EXPERIMENTAL INVESTIGATIONS ON POLARIZATION CHARACTERISTICS, ELECTRON NUMBER DENSITY AND ELECTRON COLLISIONAL FREQUENCY OF DOWN-COMING RADIO-WAVES AT OBLIQUE INCIDENCE

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ABSTRACT. The paper presents the studies carried out at Banaras on polarization characteristics of the downcoming waves of medium-wavelengths at oblique incidence. The crossed loop-aerial system of Ratcliffe and White was employed. The broadcasting stations were so chosen, that the ground waves from them did not reach Benaras.

The downcoming wave was found to be in general elliptic. It was found that only the ordinary waves with a left-handed sense of rotation were received from the medium-wave stations.

The method of transforming the observed polarization parameters from the set of axes in and at right angles to the plane of incidence, to the set of axes in and at right angles to the magnetic plane has been given.

From the polarization parameters, transformed in the above manner, the values of electron number density and electron collisional frequency have been estimated. The values of electron collisional frequency have been found to be within the expected limits for the E-layer. In the case of electron number density, the values obtained from two stations were found to be in agreement with those obtained from the normal sounding studies of Eckersley and Farmer. The discrepency in the case of the other stations is discussed.

INTRODUCTION

The experimental studies of the polarization characteristics of downcoming radio waves were initiated by Appleton and Ratcliffe (1928). They showed that the downcoming medium radio waves were in general, elliptically polarized with a left-handed sense of rotation in the northern hemisphere, as expected from the magneto-ionic theory of Appleton (1932) and Hartree (1929), where the angle between the direction of propagation and the positive direction of the earth's magnetic field was acute. The prediction of Appleton and Ratcliffe from the magneto-ionic theory, that under similar conditions of propagation the downcoming waves should acquire a right-handed sense of rotation in the southern hemisphere, was later verified and confirmed by Green (1934) in Australia. Following these workers,

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various investigators (Ratcliffe and White, 1933, Martyn and Green, 1935; Eckersley and Millington, 1939, Eckersley, 1950) carried out polarization experiments with different apparatus of their own design Though all these investigators established the general nature of polarization expected in the southern and nothern hemispheres, only Eckersley and Millington undertook a quantitative study in the medium-frequency range, by exactly finding out the ellipticity of the polarization ellipse in the downcoming wave and comparing it with the ellipticity expected from the theory for such a downcoming wave Though the results of Eckersley and Millington were in general agreement with the nature of polarization expected in the northern hemisphere, they were not able to establish conclusively the agreement of the experimental results with the theoretical values for all the stations employed for this study, excepting in the cases of only two stations The theoretican values taken by Eckersley and Millington for comparision were based on the Appleton-Hartee formulae which is true for vertical incidence only, whereas the downcoming waves were received at oblique incidence. Budden (1952), however, expressed his view that the applicability of Appleton-Hatree formulae by Eckersley and Millington was not justified for larger angles of incidence, although their results, appeared to show some agreement with the Appleton-Hartree formulae in the cases of two stations Under these circumstances, it was considered desirable to undertake a systematic study of the polarization of the downcoming waves arriving at large angles of incidence in order to examine how far the experimental results would agree with the theory Accordingly the present investigation was carried out to determine the experimental values of the polarization characteristics for oblique propagation of radio-waves through the ionosphere, as received at Banaras from the various transmitting stations situated in India and Pakistan and then to obtain the electron number density and electron collisional frequency by using the formulae given by Murty and Khastgir (1959).

The experimental technique in the present investigation was the same as that adopted by Ratcliffe and White (1933). Some advantages of this method of experimental study of the downcoming radio-wave and its suitability for the investigation of rapidly varying polarization, are mentioned below :

(i) This method is free from the assumption that is made in Appleton and Ratcliffe's frequency-change method that the polarization of the downcoming wave should remain sensibly constant for about 15 seconds or more.

(ii) With regard to the method of Eckersley and Millington, it may be mentioned that for the accurate determination of both the ratio of the axes and the tilt-angles of the polarization ellipse, it is essential to choose only those stations for which the ratio of the axes of the projected polarization ellipse lies between 0 3 and 0.6. Since in the present investigation on downcoming radio-waves coming from the different transmitting stations situated at widely different distances, it is expected to have a widely varying polarization, the method of Eckersley and Millington has not been adopted.

(iii) A definite advantage offered by Ratcliffe and White's method over other methods is that the polarization changes, even though rapid, can be followed visually and photographically

In the present investigation the polarization characteristics from five medium-wave transmitting stations were studied. All these stations studied are distant stations from which the ground waves did not reach the receiving station. All the stations with their names, wavelengths and frequencies are given in Table 1.

TABLE I

	Transmitting station	Wavelength in meters	Frequency Kc/s.	Distance from Banaras (Km.)
	Medium-wave stations			
1.	Calcutta (AIR)	300.0	1000	636
2	Dacca (Radio Pakistan)	257.1	1167	756
3	Delhi-A (AIR)	338.6	886	684
4	Hyderabad-Deccan (AIR)	411.0	730	990
5	Lahore (Radio Pakistan)	276.0	1100	1087

METHOD OF RATCLIFFE AND WHITE

EXPERIMENTAL

The study of the polarization of the downcoming radio-waves from the different broadcasting stations was made by the method of Ratcliffe and White (1933) In this method a pair of 'crossed' loop-aerials were connected through two separate *R/F* amplifier units to the opposite pairs of the deflecting plates of a cathode-ray oscillograph One of the loop aerials was placed in the direction of arrival of the downcoming wave, and the other at right angles to it The E.M.F.'s induced in these loop-aerials were then amplified equally by two suitable *R/F* amplifier units which were arranged to give linear amplification If the E.M.F.'s induced in the two loop-aerials were in phase, then the amplified voltages tuned to the frequency of the downcoming wave and applied to the pairs of deflecting plates of the cathode-ray oscillograph were also in phase and a straight line was obtained on the oscillograph screen. When there was a phase-difference between the induced E.M.F.s, an elliptic or circular pattern was obtained. The polarization characteristics of the downcoming wave were studied from the type of the patterns observed on the oscillographic screen.

The details of the equipment employed in this investigation are described below ·

'Crossed' loop-aerial system and its associated circuit connections

For working on medium wavelengths an aerial system consisting of two 'crossed' loop-aerials, 6 ft square, capable of rotation about a common vertical acis was installed on the roof of the top floor of the Physics Department building (Banaras University). The observation room is of the dimensions of $11' \times 11' \times 11'$, with 9″ thick walls and with no iron or any metal structure in it. Each loop-aerial could be tuned over a range of fequencies by means of a pair of similar ganged condensers, enclosed in a shielded metal box arranged symmetrically with respect to the loop the common plates of the ganged condensers being earthed. The output terminals from the extreme plates of the two ganged condensers, for each aerial, were connected to the input terminals of the push-pull stage of each of the two identical *R/F* amplifier units which will be described later The connecting wires from each loop to the extreme plates of the two ganged condensers, and then from the latter to the input terminals of each amplifier, were shielded and arranged symmetrically With symmetrical arrangement and with the push-pull arrangement, the antenna-effect in either loop was practically eliminated

Since the two loop-aerials were in close proximity, and since both the amplifiers were connected to the same oscillograph, it was necessary to guard against the interaction effects which would be due to the following causes

(i) Coupling between the two loop-aerials

(ii) Coupling between the two output circuits by way of electrode capacitances of the deflecting plates of the oscillograph

(iii) Coupling between the output of one amplifier and the input of the other amplifier.

Since the two loop-aerials were exactly perpendicular to each other and their associated circuits were arranged symmetrically and further since shielded wires were used from the loop-aerials to the ganged condensers and from the condensers to the amplifier units, the coupling between the two loops was eliminated. The coupling between the two output circuits was, however, found to be negligible at these frequencies with the Du Mont Oscillograph, Model 274-A used during the course of this investigation In order to prevent interaction between the output end of one amplifier and the input end of the other, the original frequency was converted into an intermediate frequency by a local osciallator after the first push-pull stage of the *R/F* amplifier unit. It was essential to employ a common oscillator in order to keep the phase relation between the two *I/F* output voltages the same as that existing between the voltages before frequency conversion. The use of a common oscillator was also an advantage as the I/F could be maintained at the same value in both the amplifiers.

Radio-frequency amplifier units

(i) *Description of the Circuits* The two similar R/F amplifier units, the purpose of which was to obtain linear amplification, sufficient to produce a good response in the oscillograph, were designed and constructed as follows :

In the first stage of each amplifier two similar pentode valves (6AC7) were used in push-pull, the control grid of each valve being connected through a suitable leak resistance to the earth. The output voltage from each of the two loop-aerials, each of which was tuned by a ganged condenser was applied to the two grids of the push-pull circuit of each amplifier unit. The necessity of employing a ganged condenser with the common plates earthed was due to the push-pull arrangement in the first stage of the amplifier unit. The primary coil of an H/F transformer joined the anodes of the two valves in the push-pull stage in the usual way. The secondary coil was tuned by a variable tuning condenser. The voltage across the tuned secondary was then applied to the grid and the filament of the pentagrid mixer (6J7). Oscillations generated in a separate circuit using a 6J5 triode valve were fed into the mixer tube in the usual way. The same oscillatory voltage was fed into the mixer tube (6L7) of the second R/F amplifier unit. The *I*/F voltage output in the plate circuit of the mixer tube was fed through an I/F amplifier (6SJ7) to the deflecting plates of the oscillograph. The whole amplifier unit was well shielded. For achieving identical performance of the two amplifiers, care was taken to use similar components in similar circuits of the two amplifier units. The biasing voltage to the control grid of the various values was obtained by means of self-biasing resistors in the cathode leads of the respective valves as shown in the circuit diagrams (Fig. 1).

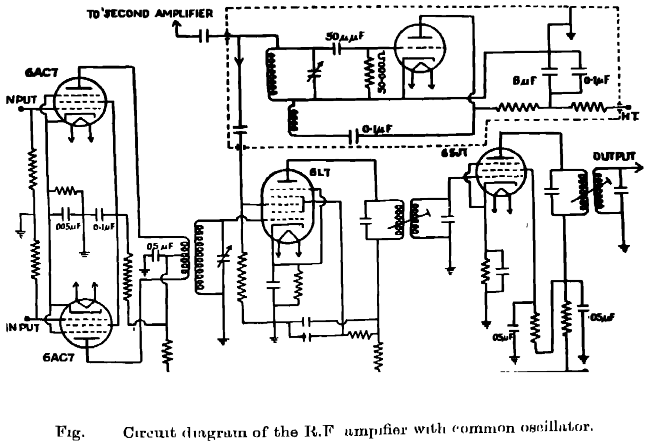


Fig. Circuit diagram of the R.F ampifier with common oscillator.

(ii) *Power Unit* : To avoid any interaction between the two R/F amplifier units through the common power supply, separate power supply units were employed for working the two R/F amplifier units. The power supply units were of the conventional type used in communication receivers, employing a full-wave rectifier valve of Type 80 and a condenser-choke-filter network to smooth the D.C. output to the desired level. The D.C. output voltage from the power unit was supplied to the various anodes and screen grids of the amplifier valves through suitable dropping resistors and by-passing capacitors.

(iii) *Chatacteristics Curves*. As the two R/F amplifier units should be identical and their amplifications should be linear, it was necessary to study the gain characteristics of the amplifier units. Employing a signal generator with a calibrated output, the gain of each amplifier unit was determined for various input voltage at required frequencies and found to be satisfactory. It was also found that the two sections of the push-pull circuit were exactly similar.

THE OSCILLOGRAPH AND PHOTOGRAPHIC ARRANGEMENTS

A Du Mont oscillograph (Type 274-A) with blue fluorescence was employed for obtaining the elliptical patterns on its fluorescent screen. The deflection sensitivities for the X and Y deflector plates were 18 rms volts/inch and 16 rms volts/inch respectively. The gains of the two R/F amplifier units were sufficient to produce good elliptic patterns on the oscilograph screen. The patterns were photographed by an Alpha Camera (f/1.8 lens) and also by a Cossor Oscillograph Camera ($f/3.5$ lens).

THEORY OF THE RECEPTION OF THE DOWNCOMING RADIO-WAVE FROM THE DISTANT BROADCAST STATIONS

For distant transmitting stations the ground waves did not reach the receiving point. Let the downcoming wave be incident at the receiving station at an angle i in the plane of incidence of the wave. The wave is reflected from the ground at the same angle. The downcoming wave is in general abnormally polarized. Let the component of the magnetic vector at right angles to the plane of incidence be $\mathbf{H}_1$ and the component in the plane of incidence be $\mathbf{H}_2$, the corresponding electric vectors being $\mathbf{E}_1$ and $\mathbf{E}_2$ respectively. The corresponding vectors for the wave reflected from the ground can be obtained by considering the ground reflection coefficients for the two components.

If E_L is the E.M.F. induced by the wave in the loop-circuit in the plane of propagation, we have,

$$E_L = \alpha\frac{dH_1}{dt}(1+K_1) = jp\beta E_1(1+K_1) \qquad \ldots \quad (1)$$

where K_1 is the reflection coefficient of the ground for the normal magnetic component H_1 and α and β are circuit constants. The value of E_L at any instant t is assumed to be of the form e^{jpt}, where p is angular frequency of the wave. The current in the loop parallel to the direction of propagation is then given by

$$i_L = jpXE_1(1+K_1) \qquad \ldots \quad (2)$$

where X = a new circuit constant Considering now the loop-aerial, the plane of which is perpendicular to the plane of propagation of the downcoming wave, the E.M.F. induced in it is given by

$$E_F = \eta \frac{dH_2}{dt}(1+K_2)\cos i = jp\rho E_2(1+K_2)\cos i \qquad (3)$$

and the current in the perpendicular loop by

$$i = jpZE_2(1+K_2)\cos i \qquad . \quad (4)$$

where η, ρ and Z are circuit constants and K_2, the ground reflection coefficient for the abnormal magnetic component of the downcoming wave The ground plan of the fields is shown in Fig. 2

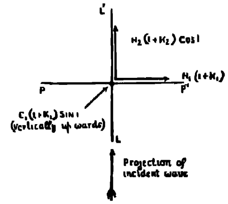


Fig. 2 Gronnd-plan of fields

Since the amplifiers are arranged to give linear amplifications, the corresponding amplified voltages e_1 and e_2 applied to the deflecting plates of the oscillograph are as follows

$$e_1 = jpX'E_1(1+K_1)e^{j(\omega-p)t} \qquad \ldots \quad (5)$$

$$e_2 = jpZ'E_2(1+K_2)e^{j(\omega-p)t} \qquad . \; . \quad (6)$$

where X' and Z' are new circuit constants, and the factor $e^{-j(\omega p)t}$ is due to the frequency change in the amplifier, ω being the angular frequency of the local oscillation and p the angular signal frequency

As in the experiments, the two loop-aerials were identical and also the two R/F amplifiers, $X' = Z'$. Further, as shown in Appendix I, $\frac{1+K_1}{1+K_2} \approx 1$ Thus the ratio of the voltages would be given by,

$$\frac{e_2}{e_1} = \frac{E_2}{E_1}\cos i \qquad \ldots \quad (7)$$

The ratio e_2/e_1 and the phase-difference ϕ between the two voltages e_2 and e_1 (which is the same as that between E_2 and E_1) were obtained from the elliptic pattern by following the procedure described later.

MEASUREMENT OF THE POLARIZATION CHARACTERISTICS FROM THE OSCILLOGRAPHIC PATTERN

Let us suppose that the Y-plates of the oscillograph are connected to the output of the amplifier unit for the loop-aerial in the direction of arrival of the down-coming waves, and the X-plates to the output of the other amplifiers unit for the other loop-aerial in the perpendicular direction. The voltage e_1 developed across the loop in the plane of incidence will give a vertical line in the Y direction and the voltage e_2 will give a horizontal line in the X direction. The resultant pattern on the oscillograph will be elliptic, circular or linear depending on the phase-difference and the amplitude-ratio of e_2 and e_1. From the elliptic pattern, (i) the ratio e_2/e_1, (ii) the phase-difference and (iii) the sense of rotation of the ellipse can be determined in the manner described below.

The ratio e_2/e_1 can be easily obtained form the sides of the rectangle which just includes the ellipse as shown in Fig. 3.

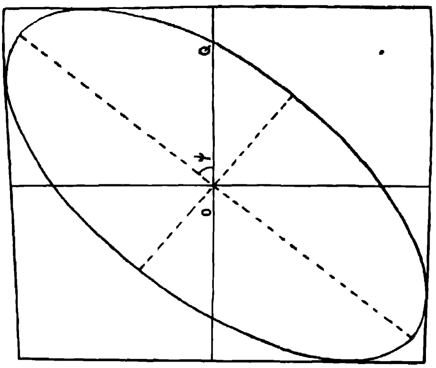


Fig. 3. Diagram of an elliptic pattern inside a rectangle.

$$\frac{e_2}{e_1} = \frac{AB}{BC} \qquad \ldots \quad (8)$$

The sine of the phase-difference can be readily determined as the ratio of the intercept of the X-axis to the maximum displacement of the spot towards the X-direction from the origin and hence will be given by

$$\sin \phi = \frac{OQ}{OX} \quad \text{(Fig. 3)} \qquad \ldots \quad (9)$$

DETERMINATION OF THE SENSE OF ROTATION

Let ϕ_1 and ϕ_2 be the phases of the amplified voltages corresponding to E_1 E_2 at any instant. Then according to our notation, the phase-difference $\phi = \phi_1 - \phi_2$. If now the phase ϕ_1 of the applied voltage e_1 corresponding to the loop-aerial in the direction of propagation and giving a linear sweep on the oscillograph in the Y-direction is decreased by increasing the capacity of the tuning condenser C_L across the parallel loop, the phase-difference ϕ becomes smaller and hence the elliptic pattern approximates more closely to a straight line. The same effect is produced on the elliptic pattern, if the phase ϕ_2 of the applied voltage e_2 corresponding to the loop-aerial in the perpendicular direction is increased by decreasing the capacity of the tuning condenser C_P across the same loop-aerial. This correspond to the condition, $0 < \phi_1 - \phi_2 < \pi/2$, provided the elliptic pattern lies in the calibration quadrant and the downcoming polarized wave has a left-handed sense of rotation (in the northern hemisphere). We have therefore the rule that if an increase of the capacity of the tuning condenser in the parallel loop-aerial or a decrease of the capacity of the tuning condenser in the perpendicular loop-aerial causes the ellipse in the calibration quadrant to approximate more closely to a straight line, the wave is to be considered left-handed and *vice versa*.

EXPERIMENTAL ADJUSTMENT

The two loop aerials and the two R/F amplifiers were first tuned to the frequency of the desired station, and the aerial connections were disconnected from the amplifier. An R/F voltage was then fed to both the amplifiers through an R/F transformer, the secondary of which was tuned by a ganged condenser. The frequency of the signal generator was adjusted to be the same as that of the desired transmitting station. As the input singal voltage was the same for both the amplifiers and as the amplifiers were already in the tuned position we would expect a straight line on the oscillograph screen. But actually, in practice, a very narrow ellipse was observed on the oscillograph screen. The narrow ellipse was reduced to a straight line by readjusting the amplifier controls. This adjustment indicates that no phase-difference was introduced by the amplifiers during the process of amplification.

The next adjustment was to get equal deflections on the oscillograph along the X and Y directions for the same voltage in the two loop-aerials. This was done by adjusting the gain controls in the amplifier units such that the straight line observed on the oscillograph screen made an angle 45° with the X direction. This 45°-alignment compensated for the unequal deflection sensitivities of the oscillograph plates along with vertical and the horizontal directions.

EXPERIMENTAL RESULTS ON THE POLARIZATION OF THE DOWNCOMING WAVE FOR OBLIQUE INCIDENCE

The amplifiers were adjusted during the day time to receive the frequency of the station under study. Receiving the signal in the day-time, the loop-aerial system was also tuned For about an hour, around sunset, the pattern on the oscillograph was found to be small and looked like a diffused patch of light. A regular polarization pattern, generally, an ellipse, was observed on the oscillograph screen from about an hour after sunset Observations were taken both visually and photographically For visual observatins, a transparent scale was attached to the oscillographic screen and the coordinates of the ends of the major axis and the intercept of the ellipse on the X-axis were noted at regular intervals of 10-15 seconds. The elliptic patterns were also photographed at suitable times.

From the observed polarization ellipse, the values of the ratio e_2/e_1 and ϕ the phase-difference were computed by the method described earlier Some respresentative polarization patterns recorded photographically after some visual notings from the five medium wave transmitting stations are shown in (Figs 4a-4e) Though the polarization pattern was in general elliptic, there was evidence of linear and circular polarization at times As the values of e_2/e_1 and ϕ were found to vary with time in a random manner, the most probable values were obtained by drawing in each case a distribution curve showing the number of times the value of e_2/e_1 and ϕ, lying within a small range was found to occur against the mean value over that range, the whole period of observation having been divided into a number of such small ranges The values of E_2/E_1 were then calculated from the corresponding values of e_2/e_1 and the most probable values of E_2/E_1, e_2/e_1 and ϕ are given in Table II

Following the procedure outlined earlier the sense of rotation of the downcoming waves, from the different transmitting stations as received at Banaras, was determined The downcoming waves from all the transmitting stations showed left-handed sense of rotation during the whole period of observation which was usually from 1900 to 2300 hrs

TABLE II

Most probable values of E_2/E_1 and e_2/e_1

Station	Meters	Distance from Banaras (Km)	Most probable values		
			e_2/e_1	E_2/E_1	ϕ
Delhi A	338 6	684	0 7	2 75	55°
Calcutta	330 0	636	0 45	1 94	60°
Hyderabad-Deccan	411.0	990	0.47	2 38	57°30′
Dacca	257.1	756	1 07	4 31	42°1l′
Lahore	276.0	1087	0 55	3.38	26°36′

In calculating the values of the critical collisional frequency ν_c, from the expression $\nu_c = \frac{p_H}{2} \frac{\sin^2 \theta'}{\cos \theta'}$, we required the earth's magnetic field H_h at the

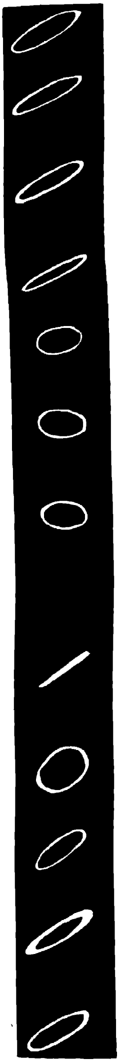
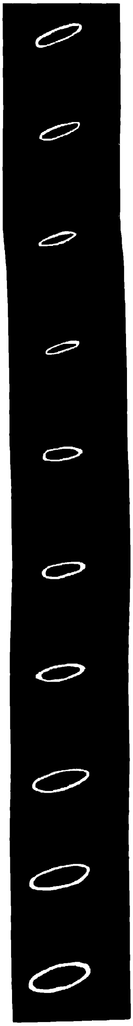
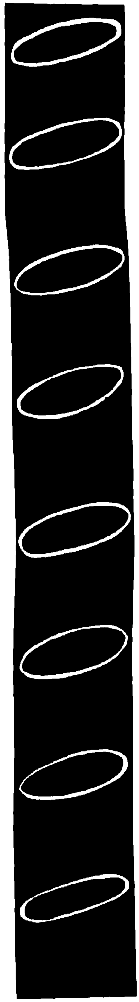


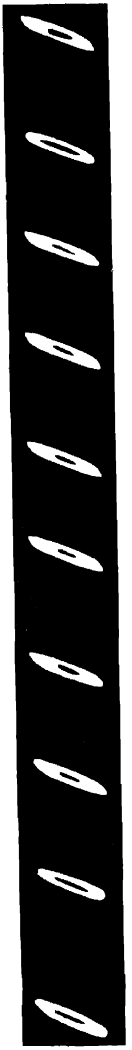
Fig 4 (a) DELHI-A (886 Kc/s). Date : 12-11-58. Time : 2034-2036 IST



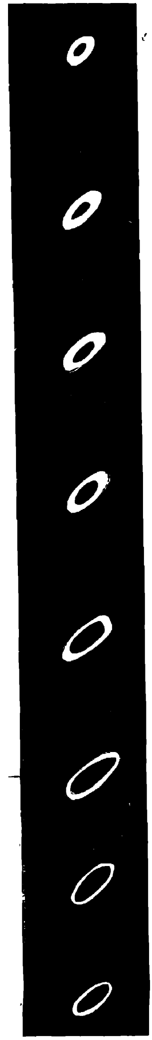
(b) CALCUTTA (1 Mc/s) Date 7-11-58 Time 2117-2119 IST



(c) HYDERABAD (730 Kc/s). Date · 16-11-58. Time . 2122-2124 IST (DECCAN)



(d) DACCA (1107 Kc/s). Date 28.10.58. Time 2050-2052 IST
(PAKISTAN)



(e) LAHORE (1100 Kc/s), Date 30-10-58. Time : 2113-2115 IST

TABLE III

Transmitting station	Wave-length in metres	Freq in Kc/s	p	D	H gauss	ι	D	ζ/θ'	H_h (gauss)	ε
Delhi	338.6	886	5.565×10^6	37°13′	0.4433	75°15′	122°24′	105°	0.4255	45°53
Calcutta	300.0	1000	6.285×10^6	33°43′	0.4384	74°12′	63°30′	59°27′	0.4199	30°11′
Hyderabad	411.0	730	4.585×10^6	28°13′	0.4222	78°36′	28° 6′	31°10′	0.04042	36°45′
Dacca	257.1	1167	7.331×10^6	34°53′	0.4420	76°36′	79°45′	73°45′	0.4233	32°43′
Lahore	276.0	1100	6.91×10^6	40°0	0.4474	80°36	12°828′	111°28′	0.4285	49°58

The values of the gyromagnetic frequency p_H the corresponding wavelength λ_H and the critical collisional frequency corresponding to the points midway between the transmitting stations and receiving stations are given in Table IV

TABLE IV

Transmitting station	$p_H = \frac{eH}{mc}$	λ_H (metres)	ν_c
Calcutta	7.38×10^6	258.4	5.38×10^6
Dacca	7.44×10^6	253.3	12.26×10^6
Delhi	7.48×10^6	252.0	13.48×10^6
Hyderabad—Deccan	7.11×10^6	265.3	1.11×10^6
Lahore	7.53×10^7	250.2	8.91×10^6

ionospheric layer and the values of the angle θ' between the direction of the down-coming wave and the positive direction of earth's magnetic field at the points midway between the various transmitting stations and the receiving station. The earth's magnetic field at a height h is given by

$$H_h = H_O\left(1-\frac{3h}{R}\right) \quad \ldots \quad (10)$$

where H_0 is the earth's magnetic field at the ground level and R, the radius of the earth.

The magnetic field H_0 at the ground level is obtained from

$$H_0 = H\sqrt{1+\tan^2 D} \quad \ldots \quad (11)$$

where H is the horizontal component of the earth's magnetic field at the ground level and D is the dip-angle

The angle between the direction of the downcoming wave and the positive direction of the earth's magnetic field is calculated from,

$$\cos\theta' = \sin D\cos i+\cos D\sin i\cos\zeta \quad \ldots \quad (12)$$

where i is the angle of incidence of the downcoming wave at the receiving station and ζ is the angle between the plane of incidence and the magnetic meridian In Table III are given the various required values of H at 90Km and θ' along with the values of D, H_0, i and ζ for the points midway between the transmitting and the receiving stations

TRANSFORMATION OF THE OBSERVED POLARIZATION ELLIPSE WHEN THE ORIGINAL SET OF AXES LYING IN AND PERPENDICULAR TO THE PLANE OF INCIDENCE IS CHANGED TO THE SET OF AXES LYING IN AND PERPENDICULAR TO THE MAGNETIC PLANE

(a) *Evaluation of the angle between the two sets of coordinate axes*

Eckersley and Millington (1939) deduced a formula for the angle ϵ between the two sets of coordinate axes : This formula is given by,

$$\tan\epsilon = \tan D\operatorname{cosec}\zeta\sin i-\cot\zeta\cos i \quad \ldots \quad (13)$$

where D is the dip-angle, i the angle of incidence of the downcoming wave at the receiver and ζ the angle between the magnetic meridian and the plane of incidence. The values of this angle calculated for the downcoming waves from the different transmitting stations are entered in Table III, along with the values of i, D, ζ and other parameters.

(b) *The amplitude-ratio and the phase-difference referred to the axes lying in and perpendicular to the magnetic meridian*

Knowing the values of the angle ϵ, the amplitude-ratio and the phase-difference, referred to the new set of axes X and Y, have been calculated from the formulae given below :

$$\rho' = \sqrt{\frac{\rho^2\cos^2\epsilon+\sin^2\epsilon+2\rho\cos\epsilon\sin\epsilon\cos\phi}{\rho^2\sin^2\epsilon+\cos^2\epsilon-2\rho\cos\epsilon\sin\epsilon\cos\phi}} \quad \ldots \quad (14)$$

and $$\tan\phi' = \frac{2\rho\sin\phi}{(1-\rho^2)\sin 2\epsilon+2\rho\cos\phi\cos 2\epsilon} \quad \ldots \quad (15)$$

where ρ and ϕ refer to the original set of X and Y The derivation of these formulae is given in Appendix-II.

The calculated values of ρ' and ϕ' obtained by substituting the experimental values of ρ and ϕ in the formulae are given in Table V

(c) *Electron number density and electron collisional frequency calculated from the polarization parameters*

The electron number density N and the electron collisional frequency ν can be calculated from the following expressions (Murty and Khastgir, 1959)

$$p' = \frac{\nu_c}{\sqrt{a'}}\cos\gamma = \frac{\nu_c\cos\gamma\sqrt{-2(\cos 2\phi+\cos 2\gamma)}}{\sin 2\phi} \quad (16)$$

$$\nu = p'\tan\gamma = \frac{\nu_c\sin\gamma\sqrt{-2(\cos 2\phi+\cos 2\gamma)}}{\sin 2\phi} \quad (17)$$

where, $$p' = p(1-p_0^2/p^2), \quad p_0^2 = 4\pi Ne^2/m, \quad a' = \frac{\nu_c^2}{\nu_c^2+p'^2}$$

$$\tan\gamma = \frac{\nu}{p'} = \frac{1+\rho^2}{1-\rho^2}\cot\phi$$

and e, m = charge and mass of an electron.

The computed values of $x = \frac{4\pi Ne^2}{mp^2} = \frac{p_0^2}{p^2}$ and also of the electron collisional frequency ν are given in Table *V*.

TABLE V

Transmitting station	ρ'	ϕ'	$x=\rho^2{}_0/\rho^2$	ν
Delhi	0 6982	34°6′	0 4141	$2\ 035\times10^6$
Calcutta	0.4888	76°32′	0 0144	2.395×10^6
Hydorabad-Deccan	0 5321	45°44′	0.8744	0.992×10^6
Dacca	0 4545	21°30′	0 6665	$9\ 279\times10^6$
Lahore	0.7042	15°0′	0.9165	6.21×10^6

DISCUSSION OF THE EXPERIMENTAL RESULTS

Electron number density from the observed polarization parameters

(a) From the values of ρ' and ϕ' obtained from the polarization parameters and given Table V, it can be seen that for the downcoming waves received at Banaras the values of x for the transmitting stations at Calcutta and Delhi are 0.014 and 0.41 respectively and the corresponding values of the electron number density are 1.750×10^2/cc and 4.33×10^3/cc These values obtained for oblique-incidence transmissions agree well with the corresponding values obtained earlier by Eckersley and Farmer (1945) for normal-incidence pulsed transmission In the case of the downcoming waves from the transmitting stations at Hyderabad (Deccan) Dacca and Lahore, the value of x varies from 0 67 to 0.92 and the corresponding electron number density ranges from 1.086×10^4/cc to $1\ 336\times10^4$/cc It may be mentioned in this connection that the values of x obtained by Roy and Verma (1955) were found to be about 0 9

There are however certain factors which may affect the calculations of x and N from the observed polarization parameters These factors are considered below.

(i) For oblique propagation the angle θ' between the direction of propagation and the positive direction of the earth's magnetic field varies continuously along the path of the wave in the ionosphere, while, in the Appleton-Hartre formulae from which the expression for the electron number density had been derived, the angle θ' is taken as constant along the path of the wave in the ionosphere The variation of θ' along the ionospheric path is expected to be large for distant transmitting stations

(ii) In calculating the angles of incidence i corresponding to the different oblique transmissions received at Banaras, the equivalent height of the E-layer has been taken to be 90 Km Any appreciable departure from the value of the equivalent height may cause perceptible variation in the value of the angle of incidence i. For very distant transmitting stations, even a small variation in the value of the E-layer height is likely to produce considerable change in the angle of incidence

(iii) The multiple reflections between the ionosphere and the earth are expected in the case of the transmissions from distant transmitting stations In our calculations, however, only the single-hop transmission has been considered.

(iv) Though the downcoming wave was found to be predominantly left-handed, the presence of the right-handed component could not altogether be ruled out

(b) *Electron collisional frequency from the observed polarization parameters*

The experimental values of the electron collisional frequency in the E-layer as given in Table V have been found to be within the limits from 1×10^6 to 10×10^6 These values lie within the expected range

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APPENDIX I

DETERMINATION OF THE VALUE OF $\frac{1+K_1}{1+K_1}$

The ground-reflection coefficients K_1 and K_2 for the normal and the abnormal components respectively are given by the expressions,

$$K_1 = \frac{\sqrt{\epsilon}\cos i - \cos r}{\sqrt{\epsilon}\cos i + \cos r} \quad \text{and} \quad K_2 = \frac{\sqrt{\epsilon}\cos r - \cos i}{\sqrt{\epsilon}\cos r + \cos i} \qquad \ldots \quad \text{(A-1)}$$

where i is the angle of incidence, r is the (complex) angle of refraction and ϵ is the complex dielectric constant of the ground given by $\epsilon = \epsilon_0 - 2j\sigma/f$, where ϵ_0 = true dielectric constant (e.s.u.), σ = conductivity (e.s.u.), and f = wave-frequency. Thus we have,

$$\frac{1+K_1}{1+K_2} = \frac{\cos i \sec r + \sqrt{\epsilon}}{\sec i \cos r + \sqrt{\epsilon}} \qquad \ldots \quad \text{(A-2)}$$

For the frequencies with which we are concerned the approximate values of ϵ_0 and σ are known to be 10 and 10^8 e.s.u respectively. The disturbing effect of ground increases as the wavelength is decreased, so that if this effect can be shown to be small for a wavelength of 257.1 m, the shortest wavelength used in the medium wave experiments, then the effect can be neglected here

For 257.1 m wavelength, ϵ is found to be equal to $(10-171.4j)$. Using this value of ϵ we get the value of $\cos r$ as approximately equal to 1 for all values of $\sin i$, thus even when $\sin i = 1$, it is found that $\cos r = 0.9999-.0029j$. Thus taking $\cos r = 1$ and putting $\sqrt{\epsilon} = 9.532-8.991j$, we find,

$$\frac{1+K_1}{1+K_2} = \frac{\cos i + 9.532-8.991j}{\sec i + 9.532-8.991j} = Re^{-j\delta} \qquad \text{... (A-3)}$$

$$\text{where} \quad R = \sqrt{\frac{(8.991)^2 + (9.532+\cos i)^2}{(8.991)^2 + (9.532+\sec i)^2}} \qquad \text{... (A-4)}$$

$$\text{and} \quad \delta = \tan^{-1}\frac{8.991}{9.532+\cos i} - \tan^{-1}\frac{8.991}{9.532+\sec i} \qquad \text{... (A-5)}$$

The values of the phase-difference δ and the amplitude-ratio R introduced by the ground reflection for various angles of incidence are shown in Fig 5. It is seen that even with an angle of incidence of 60°, the phase-difference is only

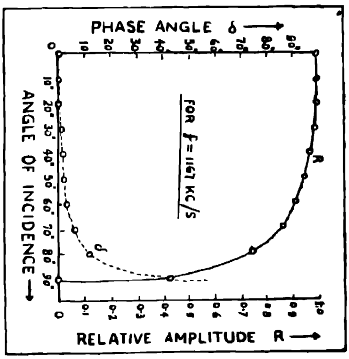

Fig. 5. Effect of ground reflection.

40 and the amplitude-ratio is 0.92 and that for the smaller angles of incidence, which occur in the experiments with the near stations, the phase-difference is less and the amplitude-ratio is more nearly unity, so that we are justified in neglecting completely the effect of the ground and in writing

$$\frac{1+K_1}{1+K} = 1 \qquad \text{(A-6)}$$

APPENDIX II

VALUES OF THE AMPLITUDE-RATIO AND PHASE-DIFFERENCE REFERRED TO THE SET OF AXES IN AND PERPENDICULAR TO THE MAGNETIC PLANE

After calculating the angle ϵ between the horizontal direction in the wave-front and the line of intersection of the magnetic plane with the wave-front from the formula given by Eckersley and Millington (1939), it is necessary to transform the experimentally observed amplitude-ratio and the phase-difference with reference to the set of axes in and perpendicular to the plane of incidence to the set of axes in and perpendicular to the magnetic plane

Let $E_1 \cos(\omega t+\phi)$ and $E_2 \cos \omega t$ be normal and the abnormal components in and perpendicular to the plane of incidence along Y-and X-directions If we now refer to the set of rectangular axes in and perpendicular to the magnetic plane along X- and Y-directions, then the normal and the abnormal electric vectors will have their respective components along the axes X- and Y-directions.

The components along X-are :

$$E_2 \cos \omega t \cos \epsilon \text{ and } E_1 \cos(\omega t+\phi) \sin \epsilon \qquad \ldots \quad \text{(A-7)}$$

and the components along Y are

$$-E_2 \cos \omega t \sin \epsilon \text{ and } E_1 \cos(\omega t+\phi) \cos \epsilon \qquad \ldots \quad \text{(A-8)}$$

The resultant electric vectors along X and Y directions will be given by

$$E_x = E_2 \cos \omega t \cos \epsilon + E_1 \cos(\omega t+\phi) \sin \epsilon \qquad \ldots \quad \text{(A-9)}$$

and

$$E_y = -E_2 \cos \omega t \sin \epsilon + E_1 \cos(\omega t+\phi) \cos \epsilon \qquad \ldots \quad \text{(A-10)}$$

Putting

$$E_x = K \cos(\omega t+\delta) \qquad \ldots \quad \text{(A-11)}$$

and

$$E_y = K' \cos(\omega t+\delta') \qquad \ldots \quad \text{(A-12)}$$

we get

$$K = \sqrt{E_2^2 \cos^2 \epsilon + E_1^2 \sin^2 \epsilon + 2E_1E_2 \sin \epsilon \cos \epsilon \cos \phi} \qquad \ldots \quad \text{(A-13)}$$

and

$$K' = \sqrt{E_2^2 \cos^2 \epsilon + E_1^2 \sin^2 \epsilon - 2E_1E_2 \sin \epsilon \cos \epsilon \cos \phi} \qquad \ldots \quad \text{(A-14)}$$

and

$$\tan \delta = \frac{E_1 \sin \phi \sin \epsilon}{E_2 \cos \epsilon + E_1 \sin \epsilon \cos \phi} \qquad \ldots \quad \text{(A-15)}$$

$$\tan \delta' = \frac{E_1 \sin \phi \cos \epsilon}{-E_2 \sin \epsilon + E_1 \cos \epsilon \cos \phi} \qquad \ldots \quad \text{(A-16)}$$

The amplitude-ratio ρ' and the phase-angle ϕ' with reference to the set of axes in and perpendicular to the magnetic plane are then given by

$$\rho' = \frac{K}{K'} = \sqrt{\frac{\sin^2\epsilon + \rho^2 \cos^2\epsilon + 2\rho \cos\epsilon \sin\epsilon \cos\phi}{\cos^2\epsilon + \rho^2 \sin^2\epsilon - 2\rho \cos\epsilon \sin\epsilon \cos\phi}} \quad \ldots \quad \text{(A-17)}$$

and

$$\tan\phi' = \frac{2\rho \sin\phi}{(1-\rho^2)\sin 2\epsilon + 2\rho \cos\phi \cos 2\epsilon} \quad \ldots \quad \text{(A-18)}$$

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